Représentations modulo p de $\mathbf{GL_2}(F)$, pour un corps p-adique F

Mod p representations of $\mathbf{GL_2}(F)$, where F is a p-adic field

Christophe BREUIL & Vytautas PASKUNAS

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Soit F une extension finie non ramifiée de $\mathbb{Q}_p$. Dans ce cours, nous associons une famille "naturelle" de représentations lisses admissibles de $\mathrm{GL}_2(F)$ sur $\overline{\mathbb{F}_p}$ à une représentation donnée de dimension 2 continue de $\mathrm{Gal}(\overline{\mathbb{Q}_p}/F)$ sur $\overline{\mathbb{F}_p}$, à condition de supposer cette dernière suffisamment "générique". Lorsque $F = \mathbb{Q}_p$, chaque famille a (essentiellement) un seul élément. Ce n'est plus le cas lorsque F n'est pas $\mathbb{Q}_p$ et la théorie dans ce cas contient une combinatoire surprenante et assez riche qui grossit "exponentiellement" avec le degré de F sur $\mathbb{Q}_p$.

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Let F be an unramified extension of $\mathbb{Q}_p$ of finite degree. In this course, we associate a "natural" family of smooth admissible representations of $\mathrm{GL}_2(F)$ over $\overline{\mathbb{F}_p}$ to a given 2-dimensional continuous representation of $\mathrm{Gal}(\overline{\mathbb{Q}_p}/F)$ over $\overline{\mathbb{F}_p}$, assuming the latter is sufficiently "generic". When $F = \mathbb{Q}_p$, each family has only one element (essentially). However, this is not the case anymore when F is not $\mathbb{Q}_p$ and the theory there reveals a quite surprising and rich combinatorics growing "exponentially" with the degree of F over $\mathbb{Q}_p$.

Algèbres de Hecke et représentations de Banach et localement analytiques des groupes de Lie p-adiques

Hecke algebras, Banach and locally analytic representations of p-adic Lie groups

Jean-François DAT & Jeremy T. TEITELBAUM

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- J.-F. Dat

  First Lecture.
  Basics on p-adic Lie groups.
  Completed group rings (Iwasawa algebras).
  Continuous Banach representation theory.

  Second Lecture.
  Banach-Hecke algebras.
  p-adic Satake isomorphism.
  "Crystalline" functoriality.

NB: The first lecture is basic for J. Teitelbaum's lectures. The second one is more independent.

- J. Teitelbaum

  First Lecture.
  1. Locally analytic functions and Distributions.
  2. Locally Analytic Representations.
  3. Frechet-Stein algebras and admissibility.

  Second Lecture.
  4. Analytic vectors in the $\mathbb{Q}_p$-analytic case.
  5. The hyperenveloping algebra and applications to analytic vectors in the L-analytic case.
  6. Brief survey of more advanced topics as time permits.

(References on page 5)

(φ, Γ)-modules et la correspondance de Langlands p-adique

(φ, Γ)-modules and the p-adic Langlands correspondence

Laurent BERGER & Pierre COLMEZ

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- L. Berger

The first half of the course will cover the required background for the construction of the p-adic Langlands correspondance.

The topics will include: (φ, Γ)-modules, p-adic representations and p-adic Hodge theory.

- P. Colmez

Le but de la seconde partie est de construire la correspondance de Langlands p-adique entre les représentations p-adiques unitaires continues superadmissibles de $\mathrm{GL}_2(\mathbb{Q}_p)$ et les représentations p-adiques de dimension 2 du groupe de Galois absolu de $\mathbb{Q}_p$.

Applications globales

Global applications

Matthew EMERTON & Florian HERZIG

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Lecture 1.
Serre-type conjecture for n-dimensional mod p Galois representations (F. Herzig) :

Background on representation theory of $\mathrm{GL}_n(\mathbb{F}_p)$ in characteristics p (Weyl modules, alcoves, irreducibles); Background on mod p Galois representations (niveaux, possible extensions (tame vs. non-tame)); Statement of the conjecture (weights) ; Evidence.

Lecture 2.
p-adic and mod p homology and cohomology (M. Emerton) :

Arithmetic quotients and p-power level towers; Cohomology, homology, various limits, etc; Basic properties: group actions, Hecke actions, admissibility; Spectral sequence for cohomology with coefficients; Poincare duality spectral sequence; Conjectures.

Lecture 3.
Serre-type conjecture, mod p cohomology, and representation theory of $\mathrm{GL}_n(\mathbb{Q}_p)$ (F. Herzig) :

Compact induction of weights; Hecke algebras (general structure, examples, action on multiplicity spaces for weights, Yoneda viewpoint); Compatibility with reduction from characteristics 0 to characteristics p; Connection to Serre-type conjecture; Incorporating Hecke eigenvalues into the conjecture (ordinary weights, possibly some remarks about non-ordinary weights); Relations with a possible mod p local and global Langlands conjecture (possibly postponed, or discussed in more detail, after Lecture 4).

Lecture 4.
The case of $\mathrm{GL}_2/\mathbb{Q}$ (M. Emerton) :

The statement of local-global compatibility; Applications (to modularity lifting theorems, possibly p-adic L-functions, ...); Sketch of proof.

Lecture 5.
We will reserve this space in case we run out of time in the earlier lectures, also for possible discussions of related topics (e.g. the possible shape of general local-global compatibility, other speculations, etc.).

(References on pages 6 and 7)

References for J.-F. Dat's lectures

- Main References:

Schneider, P.; Teitelbaum, J. *Banach space representations and Iwasawa theory.* Israel J. Math. 127 (2002), 359–380.

Schneider, P.; Teitelbaum, J. *Banach-Hecke algebras and p-adic Galois representations.* Doc. Math. 2006, Extra Vol., 631–684.

Breuil, Christophe; Schneider, Peter *First steps towards p-adic Langlands functoriality.* J. Reine Angew. Math. 610 (2007), 149–180.

- Other References:

Lazard, Michel *Groupes analytiques p-adiques.* (French) Inst. Hautes Études Sci. Publ. Math. No. 26 1965 389–603.

Coates, J.; Schneider, P.; Sujatha, R. *Modules over Iwasawa algebras.* J. Inst. Math. Jussieu 2 (2003), no. 1, 73–108.

Venjakob, Otmar *On the structure theory of the Iwasawa algebra of a p-adic Lie group.* J. Eur. Math. Soc. (JEMS) 4 (2002), no. 3, 271–311.

References for J. Teitelbaum's lectures

- Basic References:

Schneider-Teitelbaum *Lectures on Continuous and Analytic representations at Hangzhou* (Schneider home page)

Schneider-Teitelbaum *Algebras of p-adic distributions and admissible representations* (Inventiones, arxiv, Schneider home page)

Schneider-Teitelbaum *Locally analytic distributions and p-adic representation theory, with applications to L_2* (JAMS, arxiv, Schneider home page)

Schneider-Teitelbaum *$U(\mathfrak{g})$-finite locally analytic representation theory* (Rep. Theory, arxiv, Schneider home page)

Schneider-Teitelbaum *p-adic Fourier Theory* (Documenta, arxiv, Schneider home page)

- Advanced References:

M. Emerton, *Jacquet modules of locally analytic representations of p-adic reductive groups I and II.* (Ann. Sci ENS, Jussieu, Emerton home page)

Other papers of M. Emerton, esp. *p-adic L-functions and unitary completions...* (Duke, Emerton home page)

J. Kohlhaase, *Invariant distributions on p-adic analytic groups*, Muenster Preprint 410.

T. Schmidt, *Analytic vectors in continuous p-adic representations* (see arxiv:0711.2008)

T. Schmidt, *Auslander regularity of p-adic Distribution algebras* (see arxiv:math/0703464)

References for F. Herzig's lectures

A. Ash, D. Doud, D. Pollack, *Galois representations with conjectural connections to arithmetic cohomology*, Duke Mathematical Journal 112(2002), 521-579. (available at http://www2.bc.edu/~ashav/)

A. Ash, G. Stevens, *Modular forms in characteristic l and special values of their L-functions*, Duke Math. J. 53 (1986), no. 3, 849–868.

L. Barthel and R. Livne, *Irreducible modular representations of* GL_2 *of a local field*, Duke. Math. J. 75 (1994), no. 2, 261–292.

K. Buzzard, F. Diamond, A.F. Jarvis, *On Serre's conjecture for mod l Galois representations over totally real fields* (available at http://people.brandeis.edu/~fdiamond/ppt.html; esp. for Serre weights for $\mathrm{GL}_{2/\mathbb{Q}}$).

B.H. Gross, *On the Satake isomorphism*, in: Galois representations in arithmetic algebraic geometry. LMS Lecture Notes, 254, (1998), 223-238.

B.H. Gross, *Algebraic modular forms*, Israel J. Math. 113 (1999), 61-93.

F. Herzig, *The weight in a Serre-type conjecture for tame n-dimensional Galois representations* (thesis and preprint available at http://www.math.northwestern.edu/~herzig/; also for background on modular representations of $\mathrm{GL}_n(\mathbb{F}_q)$).

J.-P. Serre, *Sur les representations modulaires de degré* 2 *de* $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, Duke Math. J. 54 (1987)

References for M. Emerton's lectures

- My first lecture will include material taken from the papers:

F. Calegari, M. Emerton, *Bounds for multiplicities of unitary representations of cohomological type in spaces of cusp forms*, to appear in Annals of Math,

M. Emerton, *On the interpolation of systems of Hecke eigenvalues attached to automorphic Hecke eigenforms*, Invent. Math. 164 (2006), 1-84 (especially section 2 and subsections 4.1, 4.2, and 4.3),

as well as unpublished results due to myself and Frank Calegari.

In my lectures, I will also make reference to various classical results on the cohomology of arithmetic quotients of symmetric spaces, and its relations to the theory of automorphic forms, which can be found in:

A. Borel, N. Wallach, *Continuous cohomology, discrete subgroups, and representations of reductive groups* (2nd ed.), Math. Surveys and Monographs 67, American Math. Society, 2000,

J. Franke, *Harmonic analysis in weighted* L_2*-spaces*, Ann. Sci. Ecole. Norm. Sup. (4) 31 (1998), 181-279,

Y. Matsushima, *A formula for the Betti numbers of compact locally symmetric Riemannian manifolds*, J. Diff. Geom. 1 (1967), 99-109.

I will also rely at some points in the lecture on results related to admissible Banach representations of p-adic groups, as developed in the paper

P. Schneider, J. Teitelbaum, *Banach space representations and Iwasawa theory*, Israel J. Math. 127 (2002), 359-380,

and also on duality theory and dimension theory in non-commutative Iwasawa theory, as explained e.g. in

O. Venjakob, *On the structure theory of the Iwasawa algebra of a p-adic Lie group*, J. Eur. Math. Soc. 4 (2002), 271-311.

- My second lecture will be devoted to the statement and (if time permits) the proof of the p-adic local-global compatibility conjecture for GL_2 over $\mathbb{Q}$. It involves a specialization to and detailed study of the concepts introduced in the first lecture in the case of the group GL_2 over $\mathbb{Q}$.

  A statement and detailed discussion of the conjecture can be found in:

M. Emerton, *A local-global compatibility conjecture in the p-adic Langlands programme for* $\mathrm{GL}_{2,\mathbb{Q}}$, Pure and Applied Math. Quarterly 2 (2006), 279-393.

In discussing the conjecture and its proof, I will rely heavily on the theory of the p-adic local Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$, as developed in the lectures of Berger and Colmez. I may also make reference to some results from the preprints

M. Emerton, *Ordinary parts of admissible representations of p-adic reductive groups I, II*, available at http://www.math.northwestern.edu/~emerton